

Construction of a Massive ABJM Theory Without Higgs Superfields

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A massive Aharony-Bergman-Jafferis-Maldacena (ABJM) model in $\mathcal{N} = 1$ superspace is analysed by considering a Proca type mass term into the most general Faddeev-Popov action in a covariant gauge. The presence of mass term breaks the original BRST and anti-BRST invariance of the model. Further, the symmetry of the massive ABJM model is restored by extending the BRST and anti-BRST transformations. We show that the supergauge dependence of generating functional for connected diagrams occurs in presence of mass and ghost-anti-ghost condensates in the theory.

I. INTRODUCTION

According to the AdS/CFT correspondence, certain gauge theories in d -dimensions correspond to string/M-theory on backgrounds involving $d + 1$ dimensional AdS spaces and vice versa. The M-theory was discovered due to the fact that eleven-dimensional supergravity arises as a low-energy limit of the ten-dimensional Type IIA superstring [1]. In fact, the detailed study of $\text{AdS}_4/\text{CFT}_3$ correspondence on ABJM theory [2] can be found in Ref. [3]. The ABJM theory, a three-dimensional (3D) $\mathcal{N} = 6$ superconformal Chern-Simons theory having gauge group $U_k(N) \times U_{-k}(N)$ with bifundamental matter enjoying $SO(4)$ flavor symmetry, is dual to M-theory compactified on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$, and describes the low energy dynamics of a stack of M2 branes probing an orbifold singularity. In particular, this correspondence is justified by the planar free energy, which matches at strong coupling the classical IIA supergravity action on $\text{AdS}_4/\text{CFT}_3$ and gives the correct $N^{3/2}$ scaling for the number of degrees of freedom of the M2 brane theory [4, 5].

In M2-brane duality interpretation, the prediction that the 3D superconformal field theory should be the Chern-Simons gauge theory with maximal ($\mathcal{N} = 8$) supersymmetry was first implemented by Bagger, Lambert and Gustavsson (BLG) [6, 7]. The BLG theory uses algebraic structure known as Lie 3-algebras (and non-associative algebras). Still, the construction did not meet the desired dual to the M-theory on $\text{AdS}_4 \times S^7$ as it works only for the gauge group $SO(4)$. Further, Mukhi and Papageorgakis examined the BLG theory for multiple M2- branes and shown that when a scalar field in the 3-algebra develops a vacuum expectation value, the resulting Higgs mechanism has the novel effect of topological to dynamical gauge fields promotion [8, 9]. This novel Higgs mechanism is used to determine the leading higher-derivative corrections to the maximally supersymmetric BLG and ABJM theories. In each case, these superconformal theories are related, through the novel Higgs mechanism, to the Yang-Mills theory on D2-branes. A massive Yang-Mills theory via the Higgs mechanism, in which local gauge invariance is spontaneously broken by the Higgs field and, thus, a gauge field acquires mass, satisfies both renormalizability and physical unitarity [10]. A mass deformation of the Bagger-Lambert theory without breaking any supersymmetry is studied in Ref. [11] where a mass-deformed theory is one example of the 3D supersymmetric field theory with the so-called ‘non-central’ term whose superalgebra has been studied before [12, 13]. Seeking the importance of the massive superconformal Chern-Simons theory, we try to provide a massive construction of the ABJM theory without introducing Higgs superfield.

A non-perturbative construction of massive Yang-Mills fields without introducing the Higgs field is studied recently [14], where renormalizability and physical unitarity could not be established. A conven-

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tional argument for the violation of physical unitarity in the massive Yang-Mills theory without the Higgs field is mentioned in [15]. Specifically, it is shown that the violation of physical unitarity follows from the incomplete cancellation among unphysical modes: the scalar mode with the Faddeev-Popov ghost and antighost. The obvious reason for the violation of physical unitarity is the non-nilpotency of BRST invariance due to mass term. Subsequently, the unitarity and renormalizability for massive Yang-Mills fields in this construction have been recovered by extending the BRST symmetry [16].

The BRST symmetry is one of the important tools to handle the gauge theory consistently. The BRST symmetries for the superconformal Chern-Simons theories have been studied in recent past with various motivations [17–22]. For instance, the BRST symmetry, Slavnov-Taylor identities and Nielsen identities are derived for the ABJM theories in $N=3$ harmonic superspace and the gauge dependence of one-particle irreducible amplitudes is shown to be generated by a canonical flow with respect to the extended Slavnov-Taylor identity [19]. For a Delbourgo-Jarvis-Baulieu-Thierry-Mieg type gauge, the spontaneous breaking of the BRST symmetry occurs in the BLG theory and the responsible candidate for such spontaneous breaking is ghost-anti-ghost condensation [20]. The generalized BRST symmetry, by making the transformation parameter finite and field-dependent, also known as finite field-dependent BRST transformation [21], is discussed for the superconformal Chern-Simons theories [22].

Though original superconformal Chern-Simons theories are maximally supersymmetric, we consider a particular (gauge) sector of those supersymmetric gauge theories by using $\mathcal{N} = 1$ superfields in three dimensions for simplicity. In order to remove the redundancy in gauge degrees of freedom, the Faddeev-Popov action for the $\mathcal{N} = 1$ ABJM theory is constructed in the most general covariant gauge. The resulting Faddeev-Popov action respects absolutely anti-commuting BRST and anti-BRST transformation on Curci-Ferrari (CF) restricted surface. A Proca type mass term is added to the action, which breaks the gauge invariance. Therefore, this breaks the BRST and anti-BRST symmetries as well. To restore the symmetry, we extend the BRST and anti-BRST symmetry transformations in such a manner that these leave the massive ABJM theory invariant. But the cost we pay is that such extended transformations are not nilpotent. The responsible candidate, for breaking the nilpotency, is mass parameter M . Not only nilpotency, also the anticommutativity is lost due to M . Furthermore, we check the exactness of the gauge-fixed (originally BRST-exact) action under extended BRST and anti-BRST transformation and found that the presence of either mass M , gauge parameter α or condensate $\text{Tr}[\bar{c}c - \tilde{\bar{c}}\tilde{c}]$ breaks the extended-BRST exactness. Following the Kugo-Ojima subsidiary condition, for extended BRST and anti-BRST transformations, the gauge dependence of the generating functional of the connected Green functions is analysed. We found that the generating functional depends on the gauge parameter only if mass $M \neq 0$ and the condensate $\text{Tr}[\bar{c}c - \tilde{\bar{c}}\tilde{c}] \neq 0$. Further, the construction of the massive gauge superfield connections without Higgs superfields is made. With the help of these massive gauge superfields, the off-shell and on-shell extended BRST invariant condensates are also computed.

The presentation of this paper is as following. In section II, we recapitulate the construction of the $\mathcal{N} = 1$ ABJM theory in three dimensional superspace. The Faddeev-Popov treatment for this theory in most general covariant gauges is presented in section III. The massive theory without Higgs superfields and the extended BRST and anti-BRST symmetry are discussed in section IV. The gauge dependence of generating functional is elucidated. The construction of massive supergauge connections is presented in section V. The results with future motivations are reported in the last section.

II. ABJM THEORY: PRELIMINARIES

We discuss the preliminaries of the $\mathcal{N} = 1$ ABJM theory in 3D superspace (x^μ, θ_a) , where θ is a Grassmann spinor. We adopt the following notation for 3D superspace [23]:

$$\partial_\mu = \frac{\partial}{\partial x^\mu}, \quad \partial_a = \frac{\partial}{\partial \theta^a}. \quad (1)$$

The $\mathcal{N} = 1$ supersymmetry generators are conveniently represented in 3D superspace by $Q_a = \partial_a - i(\gamma^\mu \theta)_a \partial_\mu$, and follow $\{Q_a, Q_b\} = 2i\varepsilon_{bc}(\gamma^\mu)_a^c \partial_\mu$. The generators Q_a also satisfy $\{D_a, Q_b\} = 0$, where

$D_a = -i\partial_a + (\gamma^\mu\theta)_a\partial_\mu$ denote supersymmetrically invariant derivatives. The supercovariant derivatives can be defined by covariantizing D_a as follows:

$$\nabla_a = D_a + i\Gamma_a, \quad (2)$$

where the supergauge connection Γ_a belongs to the adjoint representation of one of the gauge groups $U(N) \times U(N)$. These supercovariant derivatives lead to the following off-shell superfield constraints: $\{\nabla_a, \nabla_b\} = -2i\nabla_{ab}$. The non-Abelian supercovariant superfield strength, Ω , related to supercovariant derivatives is computed by

$$[\nabla_a, \nabla_{bc}] = -\varepsilon_{a(b}\Omega_{c)}, \quad (3)$$

with

$$\Omega_a = \frac{1}{2}D^b D_a \Gamma_b - \frac{i}{2}[\Gamma^b, D_b \Gamma_a] - \frac{1}{6}[\Gamma^b, \{\Gamma_b, \Gamma_a\}]. \quad (4)$$

In component form, spinor superfield can be expressed as [23]

$$\Gamma_a = \xi_a + \frac{1}{2}\theta_a G + (\gamma^\mu\theta)_a A_\mu + i\theta^2 \left[\lambda_a - \frac{1}{2}(\gamma^\mu\partial_\mu\xi)_a \right]. \quad (5)$$

The action for the $\mathcal{N} = 1$ ABJM theory with the gauge group $U(N)_k \times U(N)_{-k}$ is given by

$$S = S_{\text{matter}} + S_{\text{CS}}, \quad (6)$$

where S_{CS} is the super-Chern-Simons action in an adjoint representation with the following explicit form:

$$\begin{aligned} S_{\text{CS}} = & \frac{k}{16\pi} \int d^3x d^2\theta \text{Tr} \left[i\Gamma^a \Omega_a + \frac{1}{6}\{\Gamma^a, \Gamma^b\} D_b \Gamma_a + \frac{i}{12}\{\Gamma^a, \Gamma^b\} \{\Gamma_a, \Gamma_b\} \right. \\ & \left. - i\tilde{\Gamma}^a \tilde{\Omega}_a - \frac{1}{6}\{\tilde{\Gamma}^a, \tilde{\Gamma}^b\} D_b \tilde{\Gamma}_a - \frac{i}{12}\{\tilde{\Gamma}^a, \tilde{\Gamma}^b\} \{\tilde{\Gamma}_a, \tilde{\Gamma}_b\} \right]. \end{aligned} \quad (7)$$

Here k is an integer and known as label. Corresponding to gauge groups $U(N)_k$ and $U(N)_{-k}$, we introduce different supergauge connections Γ^a and $\tilde{\Gamma}^a$, respectively. Analogously to (5), the non-Abelian supercovariant superfield strength involving $\tilde{\Gamma}^a$ is given by

$$\tilde{\Omega}_a = \frac{1}{2}D^b D_a \tilde{\Gamma}_b - \frac{i}{2}[\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a] - \frac{1}{6}[\tilde{\Gamma}^b, \{\tilde{\Gamma}_b, \tilde{\Gamma}_a\}]. \quad (8)$$

The matter action is given by [23],

$$S_{\text{matter}} = \frac{1}{4} \int d^3x d^2\theta \text{Tr} [\nabla^a X^{I\dagger} \nabla_a X_I], \quad (9)$$

where the matrix-valued complex scalar matter superfield X_I is in the bi-fundamental representation of the gauge group.

Under the supergauge transformations, the connections and matter superfields change as follows,

$$\begin{aligned} \delta\Gamma_a = \nabla_a \Lambda = D_a \Lambda + i[\Gamma_a, \Lambda], \quad \delta\tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{\Lambda} = D_a \tilde{\Lambda} + i[\tilde{\Gamma}_a, \tilde{\Lambda}], \\ \delta X^I = i(\Lambda X^I - X^I \tilde{\Lambda}), \quad \delta X^{I\dagger} = i(\tilde{\Lambda} X^{I\dagger} - X^{I\dagger} \Lambda), \end{aligned} \quad (10)$$

where Λ and $\tilde{\Lambda}$ are the gauge Lie-algebra-valued parameters.

Under these supergauge transformations, the $\mathcal{N} = 1$ ABJM action (6) is invariant. It is well-known that the redundancies in supergauge degrees of freedom are associated with the gauge symmetry. So, in the next section, we would try to remove them by quantizing the ABJM model using Faddeev-Popov method.

III. THE ABJM THEORY: BRST SYMMETRY

In order to remove unphysical supergauge degrees of freedom, we break the local supergauge invariance with the supergauge conditions: $D^a \Gamma_a = 0$ and $D^a \tilde{\Gamma}_a = 0$. Now, the effect of such conditions is incorporated in the theory by adding the following (most general) Faddeev-Popov term to the supergauge invariant action:

$$S_{\text{gf+gh}} = \int d^3x d^2\theta \text{Tr} \left[B(D^a \Gamma_a) + \frac{\alpha}{2} BB - i\frac{\alpha}{2} B[\bar{c}, c] + i\bar{c} D^a \nabla_a c - \frac{\alpha}{4} [\bar{c}, c][\bar{c}, c] \right. \\ \left. - \tilde{B}(D^a \tilde{\Gamma}_a) - \frac{\alpha}{2} \tilde{B}\tilde{B} + i\frac{\alpha}{2} \tilde{B}[\tilde{c}, \tilde{c}] - i\tilde{c} D^a \tilde{\nabla}_a \tilde{c} + \frac{\alpha}{4} [\tilde{c}, \tilde{c}][\tilde{c}, \tilde{c}] \right], \quad (11)$$

where α is a supergauge parameter. Here Nakanishi-Lautrup type superfields \bar{B} and B , $\tilde{\bar{B}}$ and \tilde{B} are related with the following CF-type restrictions :

$$\begin{aligned} \bar{B} &= -B + i[\bar{c}, c], \\ \tilde{\bar{B}} &= -\tilde{B} + i[\tilde{c}, \tilde{c}]. \end{aligned} \quad (12)$$

This can be further recast as

$$S_{\text{gf+gh}} = \int d^3x d^2\theta \text{Tr} \left[B(D^a \Gamma_a) + i\bar{c} D^a \nabla_a c + \frac{\alpha}{4} (\bar{B}\tilde{B} + BB - \tilde{\bar{B}}\tilde{\tilde{B}} - \tilde{B}\tilde{\tilde{B}}) \right. \\ \left. - \tilde{B}(D^a \tilde{\Gamma}_a) - i\tilde{c} D^a \tilde{\nabla}_a \tilde{c} \right]. \quad (13)$$

Now, the resulting effective (Faddeev-Popov) action, $S + S_{\text{gf+gh}}$, is invariant under the following sets of BRST (s_b) and anti-BRST (s_{ab}) transformations :

$$\begin{aligned} s_b \Gamma_a &= \nabla_a c, \quad s_b \tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{c}, \quad s_b c = -\frac{1}{2}[c, c], \quad s_b \tilde{c} = -\frac{1}{2}[\tilde{c}, \tilde{c}], \\ s_b \bar{c} &= iB, \quad s_b \tilde{\bar{c}} = i\tilde{B}, \quad s_b B = 0, \quad s_b \tilde{B} = 0, \quad C X^I = i c X^I - i X^I \tilde{c}, \\ s_b X^{I\dagger} &= i\tilde{c} X^{I\dagger} - i X^{I\dagger} c, \end{aligned} \quad (14)$$

$$\begin{aligned} s_{ab} \Gamma_a &= \nabla_a \bar{c}, \quad s_{ab} \tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{\bar{c}}, \quad s_{ab} \bar{c} = -\frac{1}{2}[\bar{c}, \bar{c}], \quad s_{ab} \tilde{\bar{c}} = -\frac{1}{2}[\tilde{\bar{c}}, \tilde{\bar{c}}], \\ s_{ab} c &= i\tilde{B}, \quad s_{ab} \tilde{c} = i\tilde{\bar{B}}, \quad s_{ab} \tilde{B} = 0, \quad s_{ab} \tilde{\bar{B}} = 0, \quad s_{ab} X^I = i\tilde{c} X^I - i X^I \tilde{\bar{c}}, \\ s_{ab} X^{I\dagger} &= i\tilde{\bar{c}} X^{I\dagger} - i X^{I\dagger} \tilde{c}. \end{aligned} \quad (15)$$

It is easy to verify that the above transformations are nilpotent, i.e., $(s_b)^2 = (s_{ab})^2 = 0$, as well as absolutely anti-commuting in nature with CF restrictions (12), i.e., $\{s_a, s_{ab}\} = 0$.

IV. THE MASSIVE ABJM THEORY: EXTENDED BRST SYMMETRY

To construct an effective massive ABJM theory without the Higgs superfields, we introduce following Proca-type mass term :

$$S_{\text{mass}} = M^2 \int d^3x d^2\theta \text{Tr} \left[\frac{1}{2} \Gamma_a \Gamma^a + \alpha(i\bar{c}c) - \frac{1}{2} \tilde{\Gamma}_a \tilde{\Gamma}^a - \alpha(i\tilde{\bar{c}}\tilde{c}) \right]. \quad (16)$$

With this mass term, the action results in

$$S_{\text{eff}} = S + S_{\text{gf+gh}} + S_{\text{mass}}, \quad (17)$$

which, in turn, is not invariant under the BRST transformation (14) and anti-BRST transformation (15), as

$$s_b(S_{\text{mass}}) \neq 0, \quad s_{ab}(S_{\text{mass}}) \neq 0. \quad (18)$$

To restore the invariance of the action S_{eff} , we extend the BRST transformation (14) and anti-BRST transformation (15). The extended BRST transformation (s_b^m) and anti-BRST transformation (s_{ab}^m), respectively, are given by

$$\begin{aligned} s_b^m \Gamma_a &= \nabla_a c, \quad s_b^m \tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{c}, \quad s_b^m c = -\frac{1}{2}[c, c], \quad s_b^m \tilde{c} = -\frac{1}{2}[\tilde{c}, \tilde{c}], \\ s_b^m \bar{c} &= iB, \quad s_b^m \tilde{\bar{c}} = i\tilde{B}, \quad s_b^m B = M^2 c, \quad s_b^m \tilde{B} = M^2 \tilde{c}, \\ s_b^m X^I &= i c X^I - i X^I \tilde{c}, \quad s_b^m X^{I\dagger} = i \tilde{c} X^{I\dagger} - i X^{I\dagger} c, \end{aligned} \quad (19)$$

and

$$\begin{aligned} s_{ab}^m \Gamma_a &= \nabla_a \bar{c}, \quad s_{ab}^m \tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{\bar{c}}, \quad s_{ab}^m \bar{c} = -\frac{1}{2}[\bar{c}, \bar{c}], \quad s_{ab}^m \tilde{\bar{c}} = -\frac{1}{2}[\tilde{\bar{c}}, \tilde{\bar{c}}], \\ s_{ab}^m c &= i\bar{B}, \quad s_{ab}^m \tilde{c} = i\tilde{\bar{B}}, \quad s_{ab}^m \bar{B} = -M^2 \bar{c}, \quad s_{ab}^m \tilde{\bar{B}} = -M^2 \tilde{\bar{c}}, \\ s_{ab}^m X^I &= i \bar{c} X^I - i X^I \tilde{\bar{c}}, \quad s_{ab}^m X^{I\dagger} = i \tilde{\bar{c}} X^{I\dagger} - i X^{I\dagger} \bar{c}. \end{aligned} \quad (20)$$

It is observed that the extended BRST and anti-BRST transformations leave the massive effective action (17) invariant, however the mass term is invariant under these as

$$s_b^m(S_{\text{mass}}) \neq 0, \quad s_{ab}^m(S_{\text{mass}}) \neq 0. \quad (21)$$

The Nakanishi-Lautrup superfields transform under extended BRST and anti-BRST transformations as

$$\begin{aligned} s_b^m \bar{B} &= [\bar{B}, c] - M^2 c, \quad s_b^m \tilde{\bar{B}} = [\tilde{\bar{B}}, \tilde{c}] - M^2 \tilde{c}, \\ s_{ab}^m B &= [B, \bar{c}] + M^2 \bar{c}, \quad s_{ab}^m \tilde{B} = [\tilde{B}, \tilde{\bar{c}}] + M^2 \tilde{\bar{c}}. \end{aligned} \quad (22)$$

Here, we have utilized the CF conditions (12).

To see the nilpotency of the extended BRST and anti-BRST transformations, we apply these transformations twice on each superfields and find the following non-vanishing superfields:

$$(s_b^m)^2 \bar{c} = iM^2 c, \quad (s_b^m)^2 \tilde{\bar{c}} = iM^2 \tilde{c}, \quad (s_b^m)^2 B = -M^2 [c, c], \quad (s_b^m)^2 \tilde{B} = -M^2 [\tilde{c}, \tilde{c}], \quad (23)$$

and

$$(s_{ab}^m)^2 c = iM^2 \bar{c}, \quad (s_{ab}^m)^2 \tilde{c} = iM^2 \tilde{\bar{c}}, \quad (s_{ab}^m)^2 \bar{B} = -M^2 [\bar{c}, \bar{c}], \quad (s_{ab}^m)^2 \tilde{\bar{B}} = -M^2 [\tilde{\bar{c}}, \tilde{\bar{c}}]. \quad (24)$$

It eventually confirms that the extended BRST and anti-BRST transformations are not nilpotent. However, in $M \rightarrow 0$ limit, the nilpotency of extended transformations is evident, which is obvious as these transformations in massless limit correspond to the usual BRST and anti-BRST transformations. Due to mass parameter, these extended BRST and anti-BRST transformations are not absolutely anti-commuting i.e.

$$\{s_b^m, s_{ab}^m\} \neq 0, \quad (25)$$

even on account of CF type restrictions.

The gauge-fixed action together with ghost term (11) is BRST-exact and can be expressed as

$$S_{\text{gf+gh}} = -s_b \int d^3 x d^2 \theta \text{Tr} \left[i \bar{c} \left(D^a \Gamma_a + \frac{\alpha}{2} B - i \frac{\alpha}{4} [\bar{c}, c] \right) - i \tilde{\bar{c}} \left(D^a \tilde{\Gamma}_a + \frac{\alpha}{2} \tilde{B} - i \frac{\alpha}{4} [\tilde{\bar{c}}, \tilde{c}] \right) \right]. \quad (26)$$

Moreover, this action (11) is not exact under the extended BRST and anti-BRST transformations, which is evident from the following:

$$\begin{aligned}
S_{\text{gf+gh}} &= -s_b^m \int d^3x d^2\theta \text{Tr} \left[i\bar{c} \left(D^a \Gamma_a + \frac{\alpha}{2} B - i\frac{\alpha}{4} [\bar{c}, c] \right) - i\tilde{c} \left(D^a \tilde{\Gamma}_a + \frac{\alpha}{2} \tilde{B} - i\frac{\alpha}{4} [\tilde{c}, \tilde{c}] \right) \right] \\
&\quad - i\frac{\alpha}{2} \int d^3x d^2\theta \text{Tr} \left[\bar{c} s_b^m B - \tilde{c} s_b^m \tilde{B} \right], \\
&= -s_b^m \int d^3x d^2\theta \text{Tr} \left[i\bar{c} \left(D^a \Gamma_a + \frac{\alpha}{2} B - i\frac{\alpha}{4} [\bar{c}, c] \right) - i\tilde{c} \left(D^a \tilde{\Gamma}_a + \frac{\alpha}{2} \tilde{B} - i\frac{\alpha}{4} [\tilde{c}, \tilde{c}] \right) \right] \\
&\quad - iM^2 \frac{\alpha}{2} \int d^3x d^2\theta \text{Tr} [\bar{c}c - \tilde{c}\tilde{c}], \\
&= i s_b^m s_{ab}^m \int d^3x d^2\theta \text{Tr} \left[\frac{1}{2} \Gamma^a \Gamma_a + \frac{\alpha}{2} \bar{c}c - \frac{1}{2} \tilde{\Gamma}^a \tilde{\Gamma}_a - \frac{\alpha}{2} \tilde{c}\tilde{c} \right] - iM^2 \frac{\alpha}{2} \int d^3x d^2\theta \text{Tr} [\bar{c}c - \tilde{c}\tilde{c}]. \quad (27)
\end{aligned}$$

The responsible features for this non-exactness are non-vanishing mass M , parameter α and condensate $\text{Tr} [\bar{c}c - \tilde{c}\tilde{c}]$.

The invariance of the massive ABJM action under the extended BRST transformation is justified from the following computations:

$$\begin{aligned}
s_b^m S_{\text{gf+gh}} &= \int d^3x d^2\theta \text{Tr} \left[(s_b^m B) D^a \Gamma_a + \alpha (s_b^m B) B - i\frac{\alpha}{2} (s_b^m B) [\bar{c}, c] \right. \\
&\quad \left. - (s_b^m \tilde{B}) D^a \tilde{\Gamma}_a - \alpha (s_b^m \tilde{B}) \tilde{B} + i\frac{\alpha}{2} (s_b^m \tilde{B}) [\tilde{c}, \tilde{c}] \right], \\
&= -M^2 s_b^m \int d^3x d^2\theta \text{Tr} \left[\frac{1}{2} \Gamma^a \Gamma_a + i\alpha \bar{c}c - \frac{1}{2} \tilde{\Gamma}^a \tilde{\Gamma}_a - i\alpha \tilde{c}\tilde{c} \right], \\
&= -s_b^m S_{\text{mass}}. \quad (28)
\end{aligned}$$

Consequently,

$$s_b^m (S_{\text{gf+gh}} + S_{\text{mass}}) = 0. \quad (29)$$

The classical action remains invariant under extended BRST transformation as it does not depend on Nakanishi-Lautrup superfields.

To study the supergauge dependence, we first define the vacuum functional $Z[J]$ with a source J as follows,

$$Z[J] = \int \mathcal{D}\Phi \exp \left[iS_{\text{eff}} + \int d^3x d^2\theta \text{Tr} J \mathcal{A} \right] = e^{iW[J]}, \quad (30)$$

where $\mathcal{D}\Phi$ refers to a generic functional measure, \mathcal{A} is an operator, and $W[J]$ represents the generating functional of the connected Green functions of the massive ABJM theory. The expectation value for the operator \mathcal{A} is given by

$$\langle \mathcal{A} \rangle = \frac{\delta}{\delta J} W[J] |_{J=0}. \quad (31)$$

Now, to see the dependence of $W[J]$ on α , we differentiate $W[J]$ with respect to α and obtain,

$$\frac{\partial W[J]}{\partial \alpha} = \frac{M^2}{2} \int d^3x d^2\theta \text{Tr} \langle i\bar{c}c - i\tilde{c}\tilde{c} \rangle \neq 0. \quad (32)$$

Here, the Kugo-Ojima subsidiary condition corresponding to extended BRST transformations is adopted, i.e., the conserved charges for such transformations will annihilate the physical states of the total Hilbert states. From expression (32), it is evident that $W[J]$ depends on the parameter α for mass $M \neq 0$ together

with condensate $\text{Tr}\langle i\bar{c}c - i\tilde{c}\tilde{c} \rangle \neq 0$. In the massive limit, α becomes a physical parameter defining a mass of anticommuting fields c, \bar{c}, \tilde{c} and $\tilde{\bar{c}}$ in the form αM^2 which can be seen from the equations of motion. To determine whether such ghost-anti-ghost condensation occurs or not, it is important to evaluate the effective potential for the composite operator (see, e.g., [20] for details). This result should be compared with the massless case, in which, on contrary, $W[J]$ does not depend on a gauge-fixing parameter α . This implies that, for $M = 0$, any choice of gauge-fixing parameter α gives the same generating functional $W[J]$. It is no wonder that there is no dependence on this parameter because in this limit the effective action reduces to the Faddeev-Popov action and the nilpotency of the BRST transformations is restored.

V. MASSIVE ABJM SUPERFIELDS WITHOUT HIGGS SUPERFIELDS

In this section, we construct the massive supergauge superfields \mathcal{W}_a and $\tilde{\mathcal{W}}_a$. The requirement for the physical massive vector superfields are: (i) these superfields must belong to the physical field creating a physical state with positive norm, (ii) these superfields should have the correct degrees of freedom as a massive supergauge particle, and (iii) these superfields must obey the same transformation rule as that of the original supergauge superfields.

Keeping these points in mind, \mathcal{W}_a and $\tilde{\mathcal{W}}_a$ are constructed by a nonlinear local transformations as follows,

$$\begin{aligned}\mathcal{W}_a &= \Gamma_a - \frac{1}{M^2} D_a B - \frac{1}{M^2} [\Gamma_a, B] + \frac{i}{M^2} [D_a c, \bar{c}] + \frac{i}{M^2} [[\Gamma_a, c], \bar{c}], \\ &= \Gamma_a + \frac{1}{M^2} i s_b^m s_{ab}^m \Gamma_a,\end{aligned}\tag{33}$$

$$\begin{aligned}\tilde{\mathcal{W}}_a &= \tilde{\Gamma}_a - \frac{1}{M^2} D_a \tilde{B} - \frac{1}{M^2} [\tilde{\Gamma}_a, \tilde{B}] + \frac{i}{M^2} [D_a \tilde{c}, \tilde{\bar{c}}] + \frac{i}{M^2} [[\tilde{\Gamma}_a, \tilde{c}], \tilde{\bar{c}}], \\ &= \tilde{\Gamma}_a + \frac{1}{M^2} i s_b^m s_{ab}^m \tilde{\Gamma}_a.\end{aligned}\tag{34}$$

These superfields fulfill the requirements discussed above as (i) they have the modified BRST invariance (off-mass-shell), i.e., $s_b^m \mathcal{W}_a = 0$, $s_b^m \tilde{\mathcal{W}}_a = 0$, (ii) \mathcal{W}_a and $\tilde{\mathcal{W}}^a$ are divergenceless (on-mass-shell), i.e. $D_a \mathcal{W}^a = 0$, $D_a \tilde{\mathcal{W}}^a = 0$.

With the help of expressions (33) and (34), we construct the Proca type mass terms $\frac{1}{2} M^2 \mathcal{W}_a \mathcal{W}^a$ and $\frac{1}{2} M^2 \tilde{\mathcal{W}}_a \tilde{\mathcal{W}}^a$, which are invariant under the extended BRST transformations. These will be useful for the regularization scheme to avoid divergences in the ABJM theory. Here, the (off-shell) extended BRST invariant condensates are

$$\langle \mathcal{W}_a \mathcal{W}^a \rangle, \quad \langle \tilde{\mathcal{W}}_a \tilde{\mathcal{W}}^a \rangle,\tag{35}$$

and the on-shell BRST invariant condensates are

$$\langle \frac{1}{2} \Gamma_a \Gamma^a + \alpha c \bar{c} \rangle, \quad \langle \frac{1}{2} \tilde{\Gamma}_a \tilde{\Gamma}^a + \alpha \tilde{c} \tilde{\bar{c}} \rangle.\tag{36}$$

Therefore, the massive effective ABJM action composed of massive vector fields \mathcal{W}_a and $\tilde{\mathcal{W}}^a$ is invariant under the following extended BRST and anti-BRST symmetry transformations, respectively:

$$\begin{aligned}s_b^m \mathcal{W}_a &= 0, \quad s_b^m \tilde{\mathcal{W}}_a = 0, \quad s_b^m c = -\frac{1}{2} [c, c], \quad s_b^m \bar{c} = -\frac{1}{2} [\bar{c}, \bar{c}], \\ s_b^m \bar{c} &= iB, \quad s_b^m \tilde{c} = i\tilde{B}, \quad s_b^m B = M^2 c, \quad s_b^m \tilde{B} = M^2 \tilde{c}, \\ s_b^m X^I &= i c X^I - i X^I \tilde{c}, \quad s_b^m X^{I\dagger} = i \tilde{c} X^{I\dagger} - i X^{I\dagger} c,\end{aligned}\tag{37}$$

and

$$\begin{aligned}
s_{ab}^m \mathcal{W}^a &= 0, & s_{ab}^m \tilde{\mathcal{W}}^a &= 0, & s_{ab}^m \bar{c} &= -\frac{1}{2}[\bar{c}, \bar{c}], & s_{ab}^m \tilde{\bar{c}} &= -\frac{1}{2}[\tilde{\bar{c}}, \tilde{\bar{c}}], \\
s_{ab}^m c &= i\bar{B}, & s_{ab}^m \tilde{c} &= i\tilde{\bar{B}}, & s_{ab}^m \bar{B} &= -M^2 \bar{c}, & s_{ab}^m \tilde{\bar{B}} &= -M^2 \tilde{\bar{c}}, \\
s_{ab}^m X^I &= i\bar{c}X^I - iX^I \bar{c}, & s_{ab}^m X^{I\dagger} &= i\tilde{\bar{c}}X^{I\dagger} - iX^{I\dagger} \tilde{\bar{c}}.
\end{aligned} \tag{38}$$

The present analyses will be very useful in establishing the physical unitarity and renormalizability of the ABJM theory without Higgs superfields. Though the nilpotency of the BRST symmetry leads to the physical unitarity, there is no general proof that the loss of nilpotency immediately yields the violation of physical unitarity. Therefore, even in the absence of nilpotency, there is possibility to find another way of proving physical unitarity. The physical unitarity follows from the cancellation among unphysical modes: the longitudinal and transverse modes of the gauge field together with the Faddeev-Popov ghost and antighost. Here we see that the violation of physical unitarity in the massive case follows from the incomplete cancellation among unphysical modes, as, for the massive case, the physical modes are given by a longitudinal and two transverse modes. Therefore, the remaining unphysical mode is not sufficient to cancel the ghost and antighost contributions. As a result, the elementary superfields in the original action of the ABJM model are not sufficient to respect the physical unitarity. There must be a mechanism which supplies the model with an extra bosonic mode. The non-linear superfield B is propagating in the massive case and therefore can be an important character in the cancellation in the massive case.

VI. CONCLUSIONS

In this paper, we have studied the Faddeev-Popov quantization of the $\mathcal{N} = 1$ ABJM theory in the most general covariant gauges. The absolutely anti-commuting BRST and anti-BRST transformations are also demonstrated. We have constructed a massive ABJM theory in $\mathcal{N} = 1$ superspace without Higgs superfields. The presence of mass terms break the BRST and anti-BRST symmetries of the theory. These broken symmetries are restored further by extending the BRST and anti-BRST transformations. In this context, we have found that the resulting BRST and anti-BRST symmetry transformations lose their nilpotency due to presence of mass parameter M for the superfields. These extended symmetries are not absolutely anti-commuting even on the CF restricted surface and responsible candidates are the presence of non-zero mass M , gauge parameter α and ghost-anti-ghost condensates. Further, we have studied the gauge dependence of the generating functional of connected diagrams for massive ABJM model where we adopt the Kugo-Ojima subsidiary condition corresponding to extended BRST and anti-BRST transformations. Remarkably, it is observed that the generating functional of the connected Green functions for massive ABJM model, $W[J]$, depends on the parameter α only if mass and ghost-anti-ghost condensates are present. Finally, we have constructed the massive gauge superfields without Higgs superfields which lead to the Proca mass terms. The off-shell and on-shell extended BRST invariant condensates are also evaluated. Indeed, one can show that the norm cancellation is automatically guaranteed from the Slavnov-Taylor identities if the ghost-antighost bound state exists. In this way, one can recover the physical unitarity in a nonperturbative way. We also would like to comment that to show the existence of the ghost and antighost condensate, the Nambu-Bethe-Salpeter equation is to be solved. The details about the physical unitarity and renormalizability are not discussed for the model and are the subject of future investigation.

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